CALCULATION OF THE HYDRODYNAMIC REACTIONS ON A SOLID AIRFOIL OSCILLATING IN A MOTIONLESS LIQUID

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In problems involving nonstationary detached flow past oscillating bodies an incident flow, which in the calculations plays a regularizing role in view of the fact that disturbances introduced by the body are carried away by the flow, is characteristically present. In many problems arising in the study of the hydrodynamic characteristics of wings and airfoils, however, large Strouhal numbers, when the velocity of the flow is relatively low and the amplitude of the oscillations and components of the velocities across the flow are comparatively large, must be considered. This is, as a rule, associated with the appearance of secondary detachment, which results in disagreement between the computed and experimental data.

Because of the nonlinearity and complexity of the problem it is best to develop an efficient solution within the framework of the model of an ideal incompresible medium. The use of this approach is also supported by experimental observations with visualization of the flow pattern (see, for example, [1]); these observations suggest that because the flow past the body is strongly nonstationary inertial forces will prevail over viscous forces.

This paper is devoted to a numerical investigation of the hydrodynamic characteristics of a solid airfoil undergoing translational oscillations with a finite amplitude in a motionless liquid (the Strouhal number is infinite). This author previously calculated the velocity field and the form of vortex wakes for this problem [2] including the conjugate modeling of the starting stage of formation of vortex wakes [3, 4]. Here the forces acting on the airfoil are calculated. The effect of the shape of the airfoil and the amplitude of the oscillations on the average thrust coefficient is studied.

1. Formulation of the Problem. We shall study the nonlinear problem of detached flow past an airfoil oscillating in an ideal incompressible liquid. The airfoil starts to move from a state of rest. Flow past the airfoil occurs with detachement of the fluid from the smooth free and back corner edges. The detachment is modeled with the help of vortex wakes. The point of detachment is mobile on the front edge and fixed on the back edge, and it lies at the vertex of the angle. The fluid flow outside the airfoil and the vortex wakes, which consists of tangential discontinuities of the velocity field, is assumed to be potential and the velocities of the particles of fluid are assumed to be finite.

The contour L_0 moves with the velocity $\mathbf{U}(t)$ in a direction making an angle $\theta(t)$ with the O_1x_1 axis of the stationary Cartesian coordinate system $O_1x_1y_1$. The velocity field of the particles of fluid $\mathbf{v}(x_1, y_1, t)$ is the solution of the initial- and boundary-value problem with the equations

$$\frac{d\mathbf{v}}{dt} = -\frac{1}{\rho}\nabla p, \quad \operatorname{div}\mathbf{v} = 0, \quad \operatorname{rot}\mathbf{v} = 0 \text{ outside } L; \tag{1}$$

with the boundary conditions

$$\begin{gathered}(\mathbf{v} - \mathbf{U})\mathbf{n}_0 \text{ for } (x_1, y_1) \in L_0(t),\\ [p] = 0,\ \mathbf{v}\cdot\mathbf{n} = \mathbf{v}_{\text{сл}}\cdot\mathbf{n} \text{ for } (x_1, y_1) \in L_1(t),\ L_2(t),\\ \lim\mathbf{v} = 0 \text{ for } (x_1, y_1) \to \infty,\\ |\mathbf{v}| < \infty \text{ for } (x_1, y_1) \to (x_{1M}, y_{1M}),\ (x_{1\tilde{M}}, y_{1\tilde{M}});\end{gathered} \tag{2}$$

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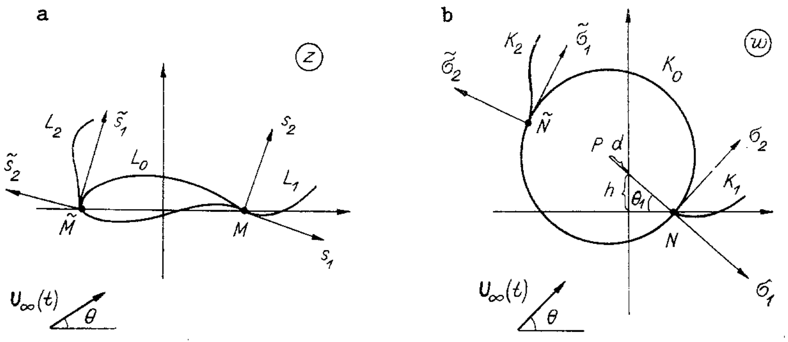

Fig. 1

and initial conditions

$$\mathbf{v}|_{t=0} = 0,\ \mathbf{U}|_{t=0} = 0,\ L|_{t=0} = L_0|_{t=0}. \tag{3}$$

here L_1 and L_2 are the contours of the wakes; $L = L_0 + L_1 + L_2$; $\mathbf{n}$ and $\mathbf{n}_0$ are unit vectors normal to the countours L and L_0; $\mathbf{v}_W$ is the velocity of a point in the wake; x_{1M} and y_{1M} are the coordinates of the sharp edge (the point M); $x_{1\tilde{M}}$ and $y_{1\tilde{M}}$ are the coordinates of the point of detachment $\tilde{M}$ on the smooth edge (Fig. 1).

The boundary conditions (2) express, respectively, the condition that the fluid cannot flow through the contour L_0, the condition that the pressure p and the velocity component normal to the wake are continuous across the contour $L_1 + L_2$, the condition that velocity field decay at infinity, and the condition that the velocity of the fluid is finite at the point of detachment. The conditions (3) express the fact that the motion starts from a state of rest and the wakes appear at the moment the motion starts. The last requirement is based on the experimental data of [1] on visualization of flow past a wing undergoing translational oscillations with a finite amplitude in a motionless liquid.

We shall study only those solutions of the problem (1)-(3) which belong to the class of functions which assume finite values at the point of detachment and near the ends of the wakes. The velocity of the flow $\mathbf{U}(t)$ past the airfoil is assumed to vary in time harmonically, and the phase of the oscillations is such that $|\mathbf{U}(0)| = 0$.

The initial- and boundary-value problem (1)-(3) reduces [5] to a Cauchy problem for the integrodifferential equations

$$(\partial\bar{\zeta}_j/\partial t)\,(t, \Gamma_j) = \bar{v}_j^0(t, \zeta_1(t, \Gamma_1), \zeta_2(t, \Gamma_2)),\ j = 1,2; \tag{4}$$

$$\zeta_j\,(t, \Gamma_j)\,|_{t=0,\Gamma_j=0} = \zeta_j^*\,(t)\,|_{t=0}, \tag{5}$$

where t is the time, j is the wake number, Γ_j is the circulation at a point in the vortex wake measured from its free end; ζ_1^* and ζ_2^* are the complex coordinates of the points of detachment of the vortex wakes $\bar{v}_j^0$ is the half-sum of the limiting values of the complex velocity of the particles of fluid as they approach the contour of discontinuity from the left and right. The complex plane $z = x + iy$ is introduced here with the help of the coordinate system Oxy (see Fig. 1) fixed on the profile; the axes of the system Oxy are parallel to the axes of the stationary system $O_1x_1y_1$.

The problem (1)-(3) is reduced to the problem (4) and (5) by solving at each moment in time a boundary-value problem of the Riemann—Hilbert type [6] in which one seeks a complex velocity function which is analytic outside the contours of the airfoil and the wakes, has a fixed jump on the lines of discontinuity, satisfies the condition of impermeability on the air foil, the kinematic and dynamic conditions on the vortex wakes, and Thompson's theorem (expressing the fact that the circulation of the velocity along a contour in the fluid that encompasses the airfoil and the contours of discontinuity vanishes), and decays at infinity and is everywhere finite. The solution of this boundary-value problem is given by the formula [6]

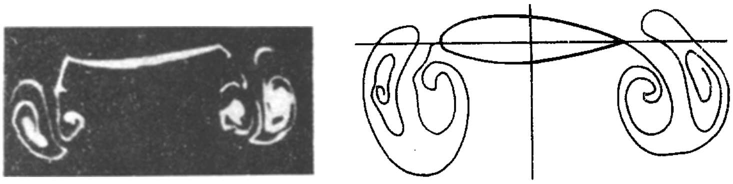

Fig. 2

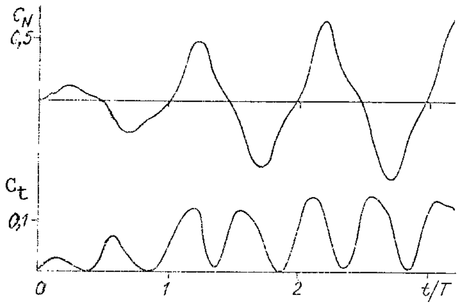

Fig. 3

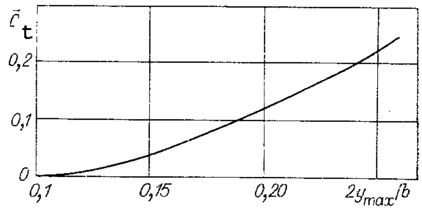

Fig. 4

$$\bar{V}(z) = \frac{\partial w}{\partial z}\left\{\bar{U}\left(1 - \frac{R^2}{(w - w_P)^2}\right) - \frac{1}{2\pi i}\sum_{j=1,2}\int_{K_j}\left(\frac{1}{\zeta_j - w} - \frac{1}{\dfrac{R^2}{\bar{w}_P - \bar{\zeta}_j} - w}\right)d\Gamma_j\right\}, \tag{6}$$

if the conformal mapping w = w (z) of the flow region into the region outside a circle of radius R (see Fig. 1) is known. Here $\bar{V}$ is the complex velocity of the particles of fluid in the z-plane in the coordinate system tied to the profile; $\bar{U}$ is the complex velocity of the fluid at infinity in the same system; and, w_P is the complex coordinate of the center P of the circle K_0 in the w-plane.

2. Computational Scheme. A detailed algorithm for solving the problem posed is studied in [2], so that we shall indicate the sequence of solution only schematically. The Cauchy problem (4) and (5) was solved numerically by modeling the wakes by a system of dicrete vortices. A family of contours which are obtained from circles (see Fig. 1) with the help of the Kármán–Trefftz mapping

$$\frac{z - \delta_1 a}{z + \delta_1 a} = \left(\frac{w - a}{w + a}\right)^{\delta_1}, \quad \delta_1 = 2 - \delta \tag{7}$$

was chosen for the airfoils in the flow. The value of a determines the chord of the airfoil, the parameter δ gives the angle on the back edge, $0 < \delta < \pi/2$, d characterizes the thickness of the airfoil, and the parameter h characterizes the curvature. It was assumed that the airfoil undergoes translational oscillations along the O_1y_1 axis according to the harmonic law $y_1 = A\cos(\omega t + \mu)$ with the frequency of oscillations ω and the amplitude A. To ensure that the motion starts smoothly it was assumed that the phase of the oscillations μ is equal to zero. The calculation of the displacement of points of the vortex wake (discrete vortices) at each moment in time was performed by Euler's scheme with the help of the formula for the velocity (6), in which with w = ζ an integral of the Cauchy type exists in the principle-value sense. The convergence of the integrals on the right side of (6) at the singular points — the points of shedding of the wakes — follows from the behavior of the intensity of the vortex wake near the edge [3] and from the fact that the curvature of the vortex wake at the point of detachment is equal to the curvature of the airfoil past which the fluid flows [4]. This relation requires that the coordinates and intensity of the shed vortices be calculated accurately.

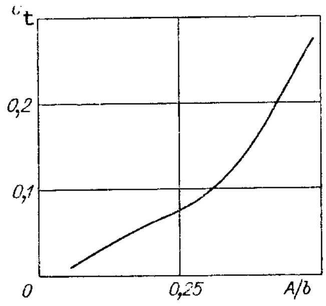

Fig. 5

The shedding of vorticity from the front smooth edge was calculated according to the following scheme. Euler's equations and the conditions on the sheet give the following relation:

$$\begin{gathered}\frac{d\tilde{\gamma}_*}{dt} = \tilde{\gamma}_* \left|\frac{\partial w}{\partial z}\right|^2 [2UR^{-1}\cos(\theta^* - \theta) - F_1 - \tilde{F}_1] + \\ + \tilde{\gamma}_* \operatorname{Re}\left[i \exp(i\theta^*) \frac{\partial^2 w}{\partial z^2}\right][2U \sin(\theta^* - \theta) + F_2 + \tilde{F}_2], \\ F_1 = \frac{1}{\pi}\int\limits_{K_1} \frac{2\sigma_1\sigma_2 + R^{-1}\sigma_1(\sigma_1^2 + \sigma_2^2)}{(\sigma_1^2+\sigma_2^2)^2}\, d\Gamma_1, \quad F_2 = \frac{1}{2\pi}\int\limits_{K_1} \frac{2\sigma_2 + R^{-1}(\sigma_1^2+\sigma_2^2)}{\sigma_1^2+\sigma_2^2}\, d\Gamma_1,\end{gathered} \tag{8}$$

which is valid for the point of shedding. In [4] it is presented for w(z) = z. Here $\tilde{\gamma}_*$ is the intensity of the vortex wake L_2 at the point of shedding; θ^* is the angular coordinate of the point of shedding in the plane w; θ is the angle of attack; $\tilde{F}_1$ and $\tilde{F}_2$ are the integrals F_1 and F_2 which the abscissa and ordinate σ_1, σ_2 of the coordinate system whose origin lies at the point M must be replaced with $\tilde{\sigma}_1$, $\tilde{\sigma}_2$ (see Fig. 1) , while the intensity Γ_1 must be replaced with Γ_2. With the help of (8) the intensity of the vortex wake $\tilde{\gamma}_*$ at the time t + Δt was determined from the formula $\tilde{\gamma}_*(t+\Delta t) = \tilde{\gamma}_*(t) + \tilde{\gamma}_*'(t)\Delta t$. The intensity of the next vortex $\Gamma(t+\Delta t) = -\tilde{\gamma}_*|\tilde{\gamma}_*|\Delta t/2$, shed from the circular edge, was calculated from the intensity $\tilde{\gamma}_*$ found, and its abscissa $\tilde{\sigma}_1 = \tilde{\gamma}_*|\partial w/\partial z|\Delta t/2$ was calculated in the moving coordinate system $\tilde{N}\tilde{\sigma}_1\tilde{\sigma}_2$ (in the plane of the conformal mapping). The other coordinate $\tilde{\sigma}_2$ of this vortex was calculated taking into account the local form of the vortex wake in a neighborhood of the point of shedding [5] with the help of the equation

$$\tilde{\sigma}_2 = -\frac{\tilde{\sigma}_1^2}{2R} + \lambda(t+\Delta t)\tilde{\sigma}_1^{5/2}. \tag{9}$$

The coefficient $\lambda(t + \Delta t)$ was determined from the position of the next to last vortex.

The position of the point of detachment at each moment in time was found from the formula $\theta^*(t+\Delta t) = \theta^*(t) + \theta^{*\prime}(t)\Delta t$ with the help of the expression for the angular velocity in the form presented in [4]:

$$\theta^{*\prime} = -R^{-1}\left[\tilde{\gamma}_*\left|\frac{\partial z}{\partial w}\right| + 2U(\theta^* - \theta) + F_2 + \tilde{F}_2\right]\left|\frac{\partial w}{\partial z}\right|^2. \tag{10}$$

The formulas (8)-(10) are the basic formulas for calculating the shedding of vorticity from the smooth front edge.

The parameters of a vortex shed at each step in time from the corner edge were calculated through the intensity of the vortex wake at the point of shedding $\gamma_*(t+\Delta t)$ using the formulas $\Gamma(t+\Delta t) = -\gamma_*|\gamma_*|\Delta t/2$, $s_1 = |\gamma_*|\Delta t/2$. The second coordinate of the vortex was found with the help of the equation for the local form of the vortex wake near the point of shedding $s_2 = \lambda(t)s_1^{3/2}$; this equation is derived analogously to Eq. (9) in [5]. The next value of the coefficient was determined, as in the case of the smooth edge, from the position of the next-to-last vortex shed from the airfoil.

The intensity γ_* required for the calculations described above was calculated with the help of the condition at the point of shedding [3]:

$$2U\sin(\theta+\theta_1)-\frac{1}{2\pi}\int\limits_{K_1}\left(\frac{2\sigma_1}{\sigma_1^2+\sigma_2^2}+R^{-1}\right)d\Gamma_1-$$
$$-\frac{1}{2\pi}\int\limits_{K_2}\left(\frac{2\tilde{\sigma}_1}{\tilde{\sigma}_1^2+\tilde{\sigma}_2^2}+R^{-1}\right)d\Gamma_2=0,$$

which expresses the fact that the component of the velocity tangent to the contour in the flow in the w-plane vanishes. Here θ_1 = arctan (h/a). It was assumed that the intensity γ of the section of wake newly shed over the time Δt is equal to the intensity sought $\gamma_*(t + \Delta t)$. The integral over this part of the contour was calculated separately taking into account the singularity of its kernel at the point of detachment and the shape of the sheet.

The numerical experiments confirmed the importance of calculating the section of the wake near the point of detachment and the fact that this section of the wake significantly affects the parameters of the vorticity shed into the flow.

An important feature of this calculation is that the starting sections of the wakes shed from the airfoil are modeled with the help of the explicit formulas presented in [3, 4]. The formulas permit constructing approximately, based on the size of the chosen starting time step, both vortex wakes shed from the front and back edges of the airfoil. The attention that must be devoted to modeling the starting stage of formation of the wakes is explained by the fact that in problems involving flow past a body which are close to linear (for example, translational motion of a thin, slightly bent airfoil under a small angle of attack) the starting parameters of the vortex wake can be chosen owing to the comparatively small number of starting parameters of the problem. In addition, the starting parameters themselves do not affect very significantly the developed regime of flow past the body. If, however, the number of geometric parameters of the airfoil and parameters in the law of motion of the airfoil is large, then the arbitrariness (choice) in giving the starting vorticity turns out to be a serious obstacle in performing systematic calculations for the purpose of determining the dependences of the hydrodynamic characteristics on the shape and law of motion of the airfoil.

The forces acting on the airfoil were calculated by integrating the pressure field on its contour. The pressure is determined by the Cauchy–Lagrange integral, which in the case of translational motion of the airfoil can be reduced, using a moving coordinate system tied to the airfoil, to the form [7] $p=-\rho\{\varphi_t+(1/2)|V|^2-(1/2)|U|^2\}+F(t)$, where $|V|$ is the modulus of the velocity in the moving coordinate system of a particle of fluid, located at the given moment t at the chosen point on the airfoil; φ_t is the time derivative of the potential of the absolute motion of the fluid in the moving coordinate system; and F(t) is an arbitrary function of time. The calculation of $|V|^2$ at the points of the airfoil is performed using the same procedures employed to calculate the velocity field. Different variants are possible for finding φ_t. The potential φ is the real part of the complex potential of the absolute motion, which for a given distribution of vorticity and position of the airfoil can be calculated exactly by integrating the formula (6) along z and transferring to absolute motion. The derivative φ_t is then found using the approximate formula $\varphi_t=(\varphi(t+\Delta t)-\varphi(t))/\Delta t$. In the numerical implementation, however, this method requires calculation of a large number of logarithms (for each point-like vortex), which appear as a result of the integration of the kernel of the integral in (6), at each moment in time, and this substantially increases the computing time.

Another method, which does not suffer from this drawback, consists of differentiating the potential of the absolute pressure, found by integrating (6) and written for discrete vortices, exactly with respect to time (in the moving coordinate system). In this case the differentiation of the logarithmic kernels of the integrals leads to expressions which are rational functions of their arguments, and the intensity of the discrete vortices does not depend on the time. The described method for finding the pressures makes it possible to use an exact formula for calculating the potential and at the same time reduce the computing time.

3. Computational Results. In the calculations the form and intensity of the vortex were studied with different values of the geometric characteristics of the airfoil (7) and amplitude of the oscillations. The data from the kinematic picture of the flow, compared with the experiments on visualization [1], are presented in detail in [2] and partially in Fig. 2. In addition, the effect of the same parameters on the hydrodynamic reactions on the airfoil and the power expended was studied. The dependences of the average thrust coefficient on the thicknening of the airfoil and the amplitude were obtained.

Figure 3 shows graphs of the behavior of the forces as a function of time. The coefficients of the normal force F_N and the thrust force F_t were calculated using the formulas $C_N = 2F_N/(\rho(\omega b)^2 b)$, $C_t = 2F_t/(\rho(\omega b)^2 b)$ (ω is the circular frequency of the oscillations of the profile and b is the chord). The curves are presented for an amplitude equal to one-fourth the chord length. The periodic regime is established after two to three periods of the oscillations. The difference of the forces in the first and second periods from their values at long times indicates that the formation of vortex wakes significantly affects the formation of hydrodynamic forces. The shift in the phases of the graphs of the normal force and the thrust force relative to the law of oscillation should also be noted. The presence of a shift is explained by the fact that the sign of the shed vorticity changes before the airfoil occupies the extreme position [2]. The magnitude of the shift depends on the shape of the airfoil and the law of oscillation.

Investigation of the effect of the solidness of the airfoil on the hydrodynamic characteristics showed that in the range of thickenings approximately from 16 to 26% of the value of the thrust coefficient average over a period of the oscillations $\bar{C}_t$ is an approximately linear function of the relative thickness of the profile $2y_{max}/b$ (Fig. 4). At the same time in the range of thickenings studied the changes in the power expended and the period-averaged modulus of the normal force do not exceed 10% (the minimum of the expended power was reached on a 16% airfoil for an amplitude to chord ratio of 0.25 and an edge angle of 0.1π). Increasing the angle on the back edge reduced the thrust force with virtually no change in the normal force and the power expended.

The graph in Fig. 5 shows the obtained quantitative dependence of the average thrust coefficient $\bar{C}_t$ on the amplitude of the oscillations for an 18% symmetric airfoil with an angle of 0.1π at the back edge. Two nearly linear sections, one of which corresponds to oscillation amplitudes up to 0.3 of the chord and the other from 0.35 to 0.5 chords, are characteristically present. The power expended increases as the amplitude increases more rapidly than does the thrust force.

In conclusion we shall present the results of a comparison of the calculations of the hydrodynamic forces with existing experimental data for translational oscillations of a wing in the mooring regime [8]. The experiment showed that thin wings with a translational oscillation law do not create a thrust force (7% profile TsAGI KV-1-7), while the thrust force for thick wings can be significant [for a 15% profile NASA-0015 the average thrust coefficient, calculated using the formula $\bar{C}_t = 2\bar{F}_t/(\rho(\omega A)^2 b)$ (A is the amplitude of the oscillations), assume the values 0.34-0.4]. The results of these calculations (Fig. 4) indicate that the average thrust force vanishes as the thickness of the profile decreases to 10%. Calculations for 15% Kármán–Trefftz profiles, approximately corresponding to the NASA-0015 profile, gave $\bar{C}_t$ = 0.35-0.47 with the same oscillation amplitude as that employed in the experiment.

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USE OF HYDRAULIC RESONANCE IN A PIPELINE WITH A GAS CAVITY TO CREATE A NONSTATIONARY JET

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UDC 532.512

High-velocity jets of liquid are widely employed in engineering for breaking down and cutting different materials. In some cases a steady jet is employed, but the results of [1] show that it is better to employ a nonstationary jet, because in this case the main mechanism of erosion of material is the high pressure of the hydraulic impact of the jet. In [2] several aspects of the creation of a nonstationary jet, emanating from a nozzle at the end of a pipe, are studied in application to hydraulic extraction of useful minerals. The oscillatory process in a pipe entirely filled with liquid is studied by the method of mathematical modeling. The nonstationary state of the jet was created either by pulsating the flow rate of the liquid at the pipe inlet or by periodically changing the cross section of the jet with the help of an oscillating valve.

It is well known that the presence of an air cavity in a liquid-filled pipe could give rise to significant oscillations of the velocity and pressure of the liquid in different nonstationary processes [3-7]. This is explained by the appearance of characteristic oscillations of a column of liquid with a frequency which is determined simultaneously by the parameters of the cavity, the liquid, and the pipe [8, 9]. Pressure oscillations during transient processes in a pipe are usually regarded as an undesirable phenomenon, so that the parameters of the air chamber are chosen so as to dampen these oscillations. At the same time there exist hydroimpact systems in which the oscillations of the liquid in a pipe without a gas cavity are specially created with the help of a valve which is periodically covered in order to obtain pressure pulses [10]. Since the presence of a gas cavity in a pipe containing liquid can lead to a significant increase in pressure [6, 7] this effect could be useful in obtaining high pressures, as pointed out in [7].

Physical-Mathematical Formulation of the Problem. In this paper we shall study the possibility of employing resonance oscillations of a liquid in a pipe with a gas cavity and a nozzle at the end (Fig. 1) to create a high-velocity pulsating jet. It is assumed that the oscillations arise as a result of modulation of the pressure $\tilde{p}_{in}(\tilde{t})$, which varies according to the law $\tilde{p}_{in}(\tilde{t}) = \tilde{p}_0 + \Delta\tilde{p}\cos(2\pi\tilde{t}/\tilde{T})$, at the pipe inlet. Here $\tilde{p}_0$ is the stationary pressure in the system, and $\Delta\tilde{p}$ and $\tilde{T}$ are the amplitude and period of the pulsation arising during pump operation. We shall study the problem in the approximation of an incompressible liquid, making the assumption that the velocity of the liquid is identical in all sections of the pipe. Neglecting the propagation time of the disturbances along the pipe in this manner will be justified if $\tilde{c}\tilde{T} \gg \tilde{L}$ ($\tilde{L}$ and $\tilde{c}$ are the length of the pipe and the velocity of propagation of the wave). We shall write the equations of motion of the liquid, the change in the volume of the gas cavity, and the adiabatic compression of the gas in the form

$$\tilde{\rho}\tilde{L}d\tilde{u}/d\tilde{t} = \tilde{p}_{in} - \tilde{p} - \lambda\tilde{L}\tilde{\rho}|\tilde{u}|\tilde{u}/2\tilde{D},$$
$$d\tilde{V}/d\tilde{t} = -\tilde{f}_0\tilde{u} + \tilde{s}_0\sqrt{2(\tilde{p}-\tilde{p}_a)/\tilde{\rho}}, \quad \tilde{p}\tilde{V}^{\gamma} = \tilde{p}_0\tilde{V}_0^{\gamma}, \qquad (1)$$

where $\tilde{u}$, $\tilde{\rho}$, and $\tilde{p}$ are the velocity, density, and pressure of the liquid at the end of the pipe; $\tilde{f}_0$ and $\tilde{s}_0$ are the cross-sectional area of the pipe and the effective area of the nozzle; $\sqrt{2(\tilde{p}-\tilde{p}_a)/\tilde{\rho}}$ is the velocity of the jet; $\tilde{p}_a$ is the atmospheric pressure; λ is the coefficient of friction at the wall; $\tilde{D}$ is the diameter of the pipe; and, γ is the adiabatic index.

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